

REGIONAL FLOOD FORMULAS USING L-MOMENTS FOR SMALL WATERSHEDS OF SONE SUBZONE OF INDIA

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ABSTRACT. Estimation of flood frequency and magnitude is essential for design of soil and water conservation measures. Data from 12 stream flow gauges within the Sone region were screened using the discordancy measure (D_i) in terms of the L-moments. Homogeneity of the region was then tested using the L-moments based heterogeneity measure, H . For computing the heterogeneity measure H , 500 simulations were performed using the four parameter Kappa distribution. Comparative regional flood frequency analysis studies were performed using the L-moments based frequency distributions: viz. Extreme value, General extreme value, Logistic, Generalized logistic, Normal, Generalized normal, Uniform, Pearson Type-III, Exponential, Generalized Pareto, Kappa, and five parameter Wakeby. Based on the L-moment ratio diagram and $|Z_i^{dist}|$ -statistic criteria, the GEV distribution was identified as the robust distribution for the study area. For estimation of floods of various return periods for gauged watersheds of the study area, a regional flood formula was developed using the L-moments based GEV distribution. Also, for estimation of floods of desired return periods for ungauged watersheds, a regional flood formula was developed by coupling the regional flood formula with the regional relationship between mean annual peak flood and watershed area.

Keywords. Design flood, L-moments, GEV distribution, Ungauged watershed, Return period.

Estimation of design flood is one of the important components of planning and management of water and land resources for sustainable development and effective implementation of soil and water conservation practices in a watershed. Information on flood magnitudes and their frequencies is needed for design of various structural and non-structural measures of soil and water conservation such as check dams, spillways, ponds, agricultural drainage systems, flood plain zoning, economic evaluation of flood protection projects etc. Pilgrim and Cordery (1992) mention that estimation of peak flows on small- to medium-sized rural drainage basins is probably the most common application of flood estimation as well as being of greatest overall economic importance. In almost all cases, no observed data are available at the design site, and little time can be spent on the estimate, precluding use of other data in the region. The authors further state that hundreds of different methods have been used for estimating floods on small drainage basins, most involving arbitrary formulas. Methods typically used for such applications are the rational method, the U.S. Soil Conservation Service method, and regional flood frequency methods. However, the choice of method depends on the applicable design criteria and availability of data.

Some of the earlier comparative flood frequency analysis studies include Landwehr et al. (1979), Wallis and Wood (1985), Hosking and Wallis (1986), Hosking and Wallis (1988), Jin and Stedinger (1989), Potter and Lettenmaier (1990), Farquharson (1992). Regional flood frequency relationships were developed based on the comparative flood frequency studies using probability weighted moment (PWM) methods, and the United States Geological Survey (USGS) method for identifying the robust distribution based on the descriptive ability and predictive ability criteria for some of the regions of India (National Institute of Hydrology, 1996; Kumar et al., 1999). L-moments are a recent development within statistics (Hosking, 1990). In a wide range of hydrologic applications, L-moments provide simple and reasonably efficient estimators of characteristics of hydrologic data and of a distribution's parameters (Stedinger et al., 1992). L-moment methods are demonstrably superior to those that have been used previously, and are now being adopted by many organizations worldwide (Hosking and Wallis, 1997). L-moments offer significant advantages over ordinary product moments, especially for environmental data sets (Zafirakou-Koulouris et al., 1998). In this study, regional flood formulas are developed based on the L-moments approach for estimation of floods of various return periods for the gauged and ungauged watersheds of Sone Subzone of India.

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L-MOMENTS APPROACH

Hosking and Wallis (1997) state that L-moments are an alternative system of describing the shapes of probability distributions. Historically they arose as modifications of the probability weighted moments (PWMs) of Greenwood et al. (1979). Probability weighted moments are defined as:

$$\beta_r = E[x\{F(x)\}^r] \quad (1)$$

which can be rewritten as:

$$\beta_r = \int_0^1 x(F)F^r dF \quad (2)$$

where $F = F(x)$ is the cumulative distribution function (CDF) for x , $x(F)$ is the inverse CDF of x evaluated at the probability F , and $r = 0, 1, 2, \dots$, is a nonnegative integer. When $r = 0$, β_0 is equal to the mean of the distribution $\mu = E[x]$.

For any distribution the r^{th} L-moment λ_r is related to the r^{th} PWM (Hosking, 1990) through

$$\lambda_{r+1} = \sum_{k=0}^r \beta_k (-1)^{r-k} \binom{r}{k} \binom{r+k}{k} \quad (3)$$

For example, the first four L-moments are related to the PWMs using:

$$\lambda_1 = \beta_0 \quad (4)$$

$$\lambda_2 = 2\beta_1 - \beta_0 \quad (5)$$

$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0 \quad (6)$$

$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0 \quad (7)$$

Hosking (1990) defined L-moment ratios as:

$$\text{L-coefficient of variation, L-CV } (\tau_2) = \lambda_2 / \lambda_1 \quad (8)$$

$$\text{L-coefficient of skewness, L-skew } (\tau_3) = \lambda_3 / \lambda_2 \quad (9)$$

$$\text{L-coefficient of kurtosis, L-kurtosis } (\tau_4) = \lambda_4 / \lambda_2 \quad (10)$$

Zafirakou-Koulouris et al. (1998) mention that like ordinary product moments, L-moments summarize the characteristics or shapes of theoretical probability distributions and observed samples. Both moment types offer measures of distributional location (mean), scale (variance), skewness (shape), and kurtosis (peakedness). The authors further mention that L-moments offer significant advantages over ordinary product moments, especially for environmental data sets, because of the following:

- L-moment ratio estimators of location, scale and shape are nearly unbiased, regardless of the probability distribution from which the observations arise (Hosking, 1990).
- L-moment ratio estimators such as L-Cv, L-skewness, and L-kurtosis can exhibit lower bias than conventional product moment ratios, especially for highly skewed samples.
- The L-moment ratio estimators of L-Cv and L-skewness do not have bounds which depend on sample size as do the ordinary product moment ratio estimators of Cv and skewness.
- L-moment estimators are linear combinations of the observations and thus are less sensitive to the largest observations in a sample than product moment estimators, which square or cube the observations.
- L-moment ratio diagrams are particularly good at identifying the distributional properties of highly skewed

data, whereas ordinary product moment diagrams are almost useless for this task (Vogel and Fennessey, 1993).

STUDY AREA AND DATA AVAILABILITY

The region defined as Sone Subzone 1(d) lies in central-eastern part of India. This region is categorized as one of the 26 hydrometeorologically homogeneous Subzones of India (Central Water Commission, 1987). Sone River is one of the major tributaries of the Ganges River flowing in the Subzone 1(d). Additional major rivers in the region include the Tons, Karmanasa, Punpun and Phalgu. The Sone Subzone 1(d) region lies between latitudes 22° 30' to 25° 45' North and longitudes 80° to 86° 15' East (fig. 1). Annual maximum peak flood data for 12 stream flow gauging sites lying in the Subzone 1(d) are available for the study. The watershed areas of these gauging sites vary from 34 to 1658 km², and the total geographical area of the Sone Subzone 1(d) is 1,28,900 km². Annual data records, ranging from 13 to 33 years, are available for the gauging sites.

METHODOLOGY AND RESULTS

Regional flood frequency analysis was performed using the various frequency distributions: viz. Extreme value (EV1), General extreme value (GEV), Logistic (LOS), Generalized logistic (GLO), Normal (NOR), Generalized normal (GNO), Uniform (UNF), Pearson Type-III (PE3), Exponential (EXP), Generalized Pareto (GPA), Kappa (KAP), and five parameter Wakeby (WAK). Parameters of the distributions were estimated using the L-moments approach. Screening of the data, discordancy measure, testing of regional homogeneity, identification of the regional distribution, and development of regional flood formulas for gauged and ungauged watersheds of Sone Subzone 1(d) are described next.

SCREENING OF DATA

The objective of screening of data is to check that the data are appropriate for performing the regional flood frequency analysis. In this study, screening of the data was performed using the L-moments based discordancy measure (D_i).

DISCORDANCY MEASURE

Hosking and Wallis (1997) defined the discordancy measure (D_i) considering if there are N sites in the group. Let $u_i = [t_2(i) \ t_3(i) \ t_4(i)]^T$ be a vector containing the sample L-moment ratios t_2 , t_3 , and t_4 values for site i , analogous to their regional values termed as τ_2 , τ_3 , and τ_4 , expressed in equations 8 to 10. T denotes transposition of a vector or matrix. Let

$$\bar{u} = N^{-1} \sum_{i=1}^N u_i \quad (11)$$

The discordancy measure for site i is defined as:

$$D_i = \frac{1}{3} N(u_i - \bar{u})^T A_m^{-1} (u_i - \bar{u}) \quad (12)$$

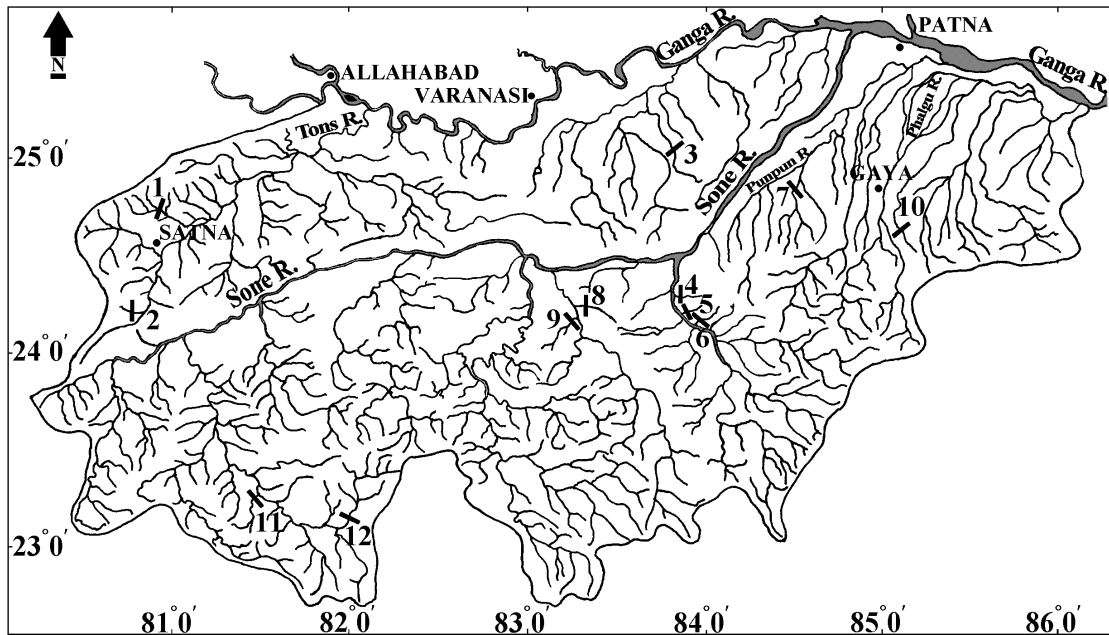


Figure 1. Index map of Sone Subzone 1(d) showing locations of the stream gauging sites.

		Legend					
		No.	Gauge Site No.	No.	Gauge Site No.	No.	Gauge Site No.
— 6	Stream gauge sites	1	1198/1	5	199	9	184
• PATNA	City/Town	2	1136/1	6	210	10	187
Tons R.	River	3	611	7	462	11	108K
		4	171	8	155	12	31

$$A_m = \sum_{i=1}^N (u_i - \bar{u})(u_i - \bar{u})^T \quad (13)$$

The site i is declared to be discordant, if D_i is greater than the critical value of the discordancy statistic D_i , given in a tabular form by Hosking and Wallis (1997).

Values of the discordancy measure were computed using the data of annual maximum peak floods of all the 12 gauging sites of the study area, and data of all the sites were found to be suitable for developing the regional flood formulas.

TEST OF REGIONAL HOMOGENEITY

For testing the regional homogeneity, a test statistic H , termed as heterogeneity measure was proposed by Hosking and Wallis (1993). It compares the inter-site variations in sample L -moments for the group of sites with what would be expected of a homogeneous region. The inter-site variation of L -moment ratio is measured as the standard deviation (V) of the at-site L -CV's weighted proportionally to the record length at each site. To establish what would be expected of a homogeneous region, simulations are used. A number of, say 500, data regions are generated based on the regional weighted average statistics using a four parameter distribution e.g. Kappa distribution. The inter-site variation of each generated region is computed and the mean (μ_v) and standard deviation (σ_v) of the computed inter-site variation is obtained. Then, heterogeneity measure H is computed as:

$$H = \frac{V - \mu_v}{\sigma_v} \quad (14)$$

The criteria established by Hosking and Wallis (1993) for assessing heterogeneity of a region is as follows.

If $H < 1$ Region is acceptably homogeneous.

If $1 \leq H < 2$ Region is possibly heterogeneous.

If $H \geq 2$ Region is definitely heterogeneous.

The heterogeneity measure (H), computed using the data of 12 gauging sites of the Subzone 1(d) was found to be greater than 1.0. Based on the statistical properties, one by one, two sites of the region were excluded until a H value less than 1.0 was obtained. The value of heterogeneity measure computed by carrying out 500 simulations using the Kappa distribution based on the data of 10 sites is obtained as $H = 0.97$. Since the heterogeneity measure value, based on the heterogeneity measure criteria, is less than one, the region comprising of 10 gauging sites was treated as a homogeneous region. The details of watershed data and statistical parameters including the discordancy measure, for the 10 gauging sites are given in table 1. It is observed that the D_i values for the 10 sites vary from 0.35 to 2.12, all of which are less than the critical D_i value of 2.491 (Hosking and Wallis, 1997). Hence, data of these 10 sites were used for development of regional flood formulas for the Sone Subzone 1(d).

IDENTIFICATION OF REGIONAL FREQUENCY DISTRIBUTION

The choice of an appropriate frequency distribution for a homogeneous region is made by comparing the moments of the distributions to the average moments statistics from regional data. The objective is to identify a distribution that best fits the observed data. The best fit is determined by how well the L -Skewness and L -Kurtosis of the fitted distribution match the regional average L -Skewness and L -Kurtosis of

Table 1. Watershed data and statistical parameters for 10 gauging sites of Sone Subzone 1(d).

Stream Gauge Site No.	Watershed Area (km ²)	Mean Annual Peak Flood (m ³ /s)	Standard Deviation (m ³ /s)	Coefficient of Variation	Coefficient of Skewness	Sample Size (years)	Discordancy Measure (D _i)
1198/1	341	224.26	172.35	0.769	1.186	31	0.37
1136/1	158	166.85	76.70	0.460	1.409	20	2.12
611	440	201.48	238.31	1.183	2.459	29	1.09
171	373	203.97	203.25	0.996	2.274	33	1.87
462	517	130.22	97.89	0.752	0.765	23	1.17
184	249	337.13	247.29	0.734	1.894	24	0.54
155	181	235.38	252.18	1.071	2.476	24	0.81
187	1658	404.89	243.86	0.602	0.892	18	0.35
108 K	279	269.06	152.25	0.566	0.387	18	1.12
31	812	584.42	613.02	1.049	2.714	12	0.55

the observed data. In this study, the L-moment ratio diagram and $|Z_i^{dist}|$ -statistic are used as the best fit criteria for identifying the regional distribution. L-moment ratio diagrams compare sample estimates of the dimensionless L-moment ratios with their theoretical counterparts (Zafira-kou-Koulouris et al., 1998). In the L-moment ratio diagram (fig. 2), the point defined by the regional average values of L-skewness i.e. $\tau_3 = 0.3481$ and L-kurtosis i.e. $\tau_4 = 0.2334$, lies closest to the GEV distribution.

The goodness-of-fit measure for a distribution, Z_i^{dist} -statistic defined by Hosking and Wallis (1993), is expressed as:

$$Z_i^{dist} = \frac{\left(\frac{-R}{\tau_i} - \tau_i^{dist} \right)}{\sigma_i^{dist}} \quad (15)$$

where $\bar{\tau}_i^R$ is the weighted regional average of L-moment statistic i , τ_i^{dist} and σ_i^{dist} are the simulated regional average and standard deviation of L-moment statistics i , respectively,

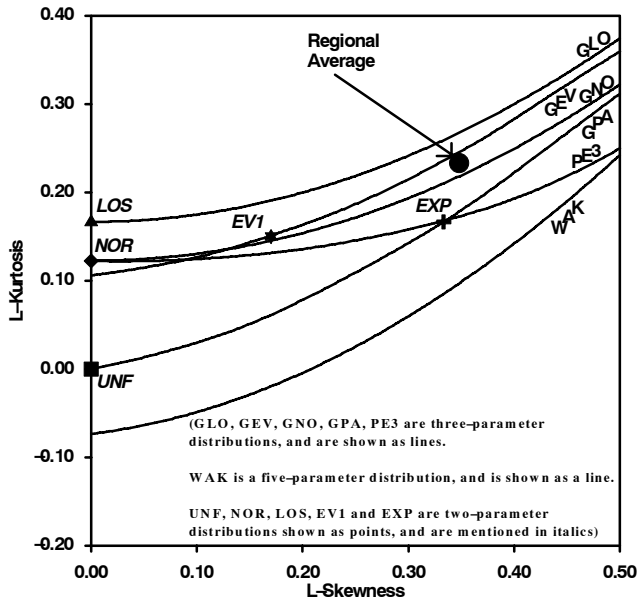


Figure 2. L-moment ratio diagram for Sone Subzone 1(d) for various distributions: Generalized Logistic (GLO); General Extreme Value (GEV); Generalized Normal (GNO); Generalized Pareto (GPA); Pearson Type-III (PE3); Wakeby (WAK); Uniform (UNF); Normal (NOR); Logistic (LOS); Extreme Value (EV1); and Exponential (EXP).

for a given distribution. The fit is considered to be adequate if $|Z_i^{dist}|$ -statistic is sufficiently close to zero, a reasonable criterion being $|Z_i^{dist}|$ -statistic less than 1.64.

The Z_i^{dist} -statistic for the various three parameter distributions is given in table 2. It is observed that the $|Z_i^{dist}|$ -statistic values are lower than 1.64 for the four distributions viz. GEV, GNO, GLO and GPA. Further, the $|Z_i^{dist}|$ -statistic is found to be the lowest for GEV distribution, i.e. 0.13; which is very close to 0.0. Thus, based on the L-moment ratio diagram as well as $|Z_i^{dist}|$ -statistic criteria, the GEV distribution is identified as the robust distribution for the Sone Subzone 1(d).

The values of regional parameters for the various distributions which have $|Z_i^{dist}|$ -statistic value less than 1.64 are given in table 3.

DEVELOPMENT OF REGIONAL FLOOD FORMULA FOR GAUGED WATERSHEDS

The GEV distribution was identified as the robust distribution for the study area; therefore, regional flood formulas have been developed using this distribution. The form of the regional formula for a GEV distribution is expressed as:

$$\frac{Q_T}{Q} = \xi + \alpha \left[1 - \left\{ -\ln \left(1 - \frac{1}{T} \right) \right\}^k \right] / k \quad (16)$$

Table 2. Z_i^{dist} -statistic for various distributions for Sone Subzone 1(d).

S. No.	Distribution	Z_i^{dist} -statistic
1	GEV	0.13
2	GNO	-0.57
3	GLO	0.70
4	GPA	-1.59
5	PE3	-1.76

Table 3. Regional parameters for the various distributions for Sone Subzone 1(d).

Distribution	Parameters of the Distribution		
GEV	$\xi = 0.597$	$\alpha = 0.439$	$k = -0.260$
GNO	$\xi = 0.754$	$\alpha = 0.584$	$k = -0.734$
GLO	$\xi = 0.915$	$\alpha = 0.308$	$k = -0.164$
GPA	$\xi = 0.188$	$\alpha = 0.786$	$k = -0.033$

Here, Q_T is T-year return period flood estimate, \bar{Q} is the mean annual maximum peak flood of the watershed, ξ , α , and k are the parameters of the GEV distribution.

The values of regional parameters of the GEV distribution for Subzone 1(d) are: $\xi = 0.597$, $\alpha = 0.439$ and $k = -0.260$ (table 3). Substituting values for these regional parameters in equation 16, the regional flood formula for estimation of floods of various return periods for the gauged watersheds of Subzone 1(d) is expressed as:

$$Q_T = \left[-1.091 + 1.688 \left\{ -\ln \left(1 - \frac{1}{T} \right) \right\}^{-0.260} \right] \times \bar{Q} \quad (17)$$

The above regional flood formula (eq. 17) may be used for estimation of floods of desired return periods for small to moderate size gauged watersheds of Subzone 1(d). Alternatively, floods of various return periods may also be computed by multiplying the mean annual peak flood of the watershed (\bar{Q}) by the corresponding value of growth factors (Q_T/\bar{Q}), which are developed using equation 17, and are given in table 4.

Similarly, substituting the values of regional parameters of GNO, GLO, and GPA distributions (table 3), having $|Z_1^{\text{dist}}|$ –statistic less than 1.64 (table 2) in their respective equations, which are available in literature (e.g. Hosking and Wallis, 1997), the growth factors were also developed for these distributions. Comparison of the growth factors developed for GEV, GNO, GLO, and GPA distributions is shown in figure 3.

DEVELOPMENT OF REGIONAL RELATIONSHIP BETWEEN MEAN ANNUAL PEAK FLOOD AND WATERSHED AREA

For estimation of T-year return period flood at a site, the estimate for mean annual peak flood is required. For ungauged watersheds at a site, a mean cannot be computed in absence of the observed flow data. In such a situation, a relationship between the mean annual peak flood of gauged watersheds in the region and their pertinent physiographic and climatic characteristics is needed for estimation of the mean annual peak flood. Since watershed areas of the 10 gauging sites of Sone Subzone 1(d) were the only physiographic characteristics available, the following regional relationship was developed in terms of watershed area for estimation of mean annual peak floods for ungauged watersheds:

$$\bar{Q} = 39.45(A)^{0.311} \quad (18)$$

Here, A is the watershed area (in km^2) and \bar{Q} is the mean annual peak flood (m^3/s). This relationship is developed based on regression analysis, using the least squares approach. For this relationship, correlation coefficient (r) is 0.50 and standard error of the estimates (SE) is 0.40.

Table 4. Values of growth factors (Q_T/\bar{Q}) for GEV distribution for Sone Subzone 1(d).

Return Period	2	5	10	25	50	100	200	500	1000
Growth Factors	0.766	1.402	1.939	2.786	3.563	4.489	5.594	7.393	9.068

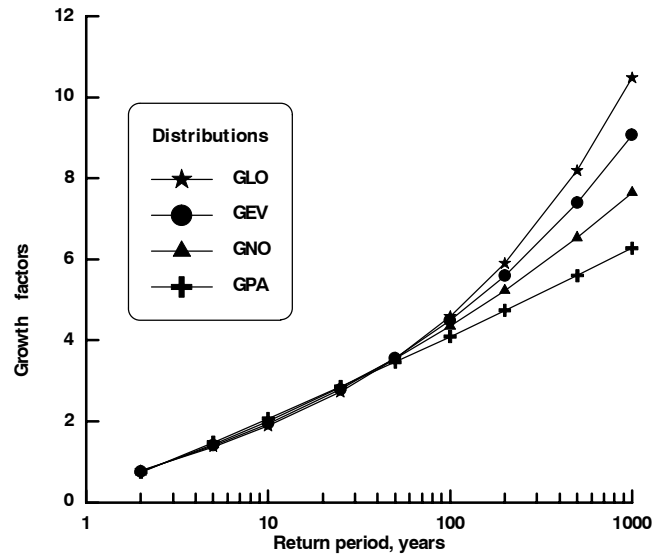


Figure 3. Comparison of growth factors for the various distributions for Sone Subzone 1(d).

DEVELOPMENT OF REGIONAL FLOOD FORMULA FOR UNGAUGED WATERSHEDS

For development of regional flood formula for estimation of floods of various return periods for ungauged watersheds, the regional flood formula given in equation 17 was coupled with the regional relationship between mean annual peak flood and watershed area, given in equation 18. The following regional formula was developed.

$$Q_T = \left[-43.040 + 66.592 \left\{ -\ln \left(1 - \frac{1}{T} \right) \right\}^{-0.26} \right] A^{0.311} \quad (19)$$

where Q_T is the flood estimate (m^3/s) for T year return period, and A is watershed area (km^2).

The above regional flood formula (eq. 19), its tabular form (table 5), or graphical representation (fig. 4) may be used for estimation of floods of desired return periods for ungauged watersheds of the Sone Subzone 1(d).

CONCLUSIONS

On the basis of this study, the following conclusions are drawn.

- Screening of the data and the regional homogeneity test reveal that the data of 10 out of the 12 gauging sites of Subzone 1(d) constitute a homogeneous region.
- Various distributions viz. EV1, GEV, LOS, GLO, NOR, GNO, UNF, PE3, EXP, GPA, KAP, and WAK have been employed. Regional parameters of the distributions have been estimated using the L-moments approach. Based on the L-moment ratio diagram and $|Z_1^{\text{dist}}|$ –statistic criteria, GEV distribution has been identified as robust distribution for the study area.

Table 5. Floods of various return periods for different watershed areas of Sone Subzone 1(d).

Watershed Area (km ²)	Return Periods (years)								
	2	5	10	25	50	100	200	500	1000
10	62	113	157	225	288	362	452	597	732
20	77	140	194	279	357	450	560	740	908
30	87	159	220	317	405	510	636	840	1030
40	95	174	241	346	443	558	695	919	1127
50	102	187	258	371	475	598	745	985	1208
60	108	198	273	393	502	633	788	1042	1278
70	113	207	287	412	527	664	827	1093	1341
80	118	216	299	429	549	692	862	1140	1398
90	122	224	310	445	570	718	894	1182	1450
100	127	232	320	460	589	742	924	1221	1498
150	144	263	363	522	668	841	1048	1386	1700
200	157	287	397	571	730	920	1147	1515	1859
250	168	308	426	612	783	986	1229	1624	1992
300	178	326	451	648	828	1044	1301	1719	2108
350	187	342	473	680	869	1095	1365	1803	2212
400	195	356	493	708	906	1141	1422	1880	2306
450	202	370	511	735	940	1184	1475	1950	2392
500	209	382	528	759	971	1223	1525	2015	2471
600	221	404	559	804	1028	1295	1614	2132	2616
700	232	424	587	843	1078	1358	1693	2237	2744
800	242	442	612	879	1124	1416	1765	2332	2860
900	251	459	634	912	1166	1469	1830	2419	2967
1000	259	474	656	942	1205	1518	1891	2500	3066
1200	274	502	694	997	1275	1606	2002	2645	3245
1400	288	526	728	1046	1338	1685	2100	2775	3404
1600	300	549	759	1090	1394	1757	2189	2893	3548
1800	311	569	787	1131	1446	1822	2271	3001	3681
2000	321	588	813	1169	1494	1883	2346	3101	3803

- For estimation of floods of various return periods for gauged watersheds of the study area, either the developed

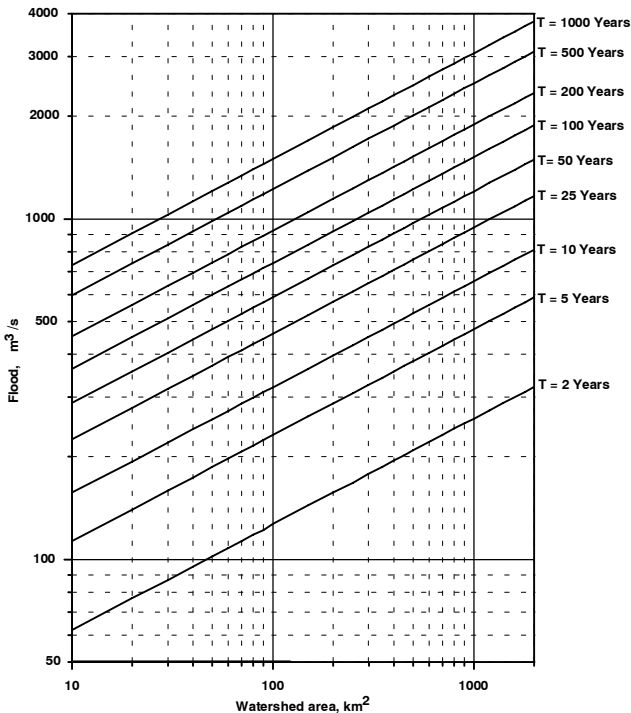


Figure 4. Variation of floods of various return periods with watershed area for Sone Subzone 1(d).

regional flood formula may be used or the mean annual peak flood of the watershed may be multiplied by corresponding values of the growth factors.

- For estimation of floods of desired return periods for ungauged watersheds of the study area, the regional flood formula developed for ungauged watersheds, its tabular form or graphical representation may be used.
- Since the regional flood formulas have been developed using the data of watersheds ranging from 158 to 1658 km² in area, these formulas may be expected to provide estimates of floods of various return periods for the watersheds of Subzone 1(d), lying nearly in the same range of land area, as those of the input data.
- For the relationship between mean annual peak flood and watershed area, the correlation coefficient is 0.50 and standard error of estimates is 0.40. However, the regional flood formulas may be refined for obtaining more accurate flood frequency estimates, when data for additional gauging sites become available and watershed and physiographic characteristics other than watershed area are also used for development of the regional flood formulas.

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